

# Electronic structure of vortices pinned by columnar defects in $p_x \pm ip_y$ superconductors.

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The electronic structure of a vortex pinned by an insulating columnar inclusion in a type-II chiral  $p_x \pm ip_y$  superconductor is studied within the Bogolubov-de Gennes theory. The structure of the anomalous spectral branch is shown to be strongly affected by the mutual orientations of the angular momenta of the center of mass and the relative motion of the two electrons in the Cooper pair. Being only slightly perturbed by the scattering at the defect for the zero sum of these angular momenta the anomalous spectral branch appears to change dramatically in the absence of such compensation. In the latter case the defect presence changes the anomalous branch slope sign at the Fermi level resulting in the quasiparticle angular momenta inversion at the positive energies and the impact parameters smaller than the defect radius. The experimentally observable consequences for the scanning tunneling microscopy characteristics and high-frequency field response are discussed.

## I. INTRODUCTION

The experimental search for superconducting compounds with unconventional order parameters remains a "hot topic" in the condensed matter community during the recent decades. Such activity is certainly accompanied by a lot of theoretical works aimed to suggest reliable tests probing the gap anisotropy at the Fermi surface. These tests are usually based either on the gap nodes presence at the Fermi surface or on the order parameter peculiar phase structure in the momentum space. The latter group of suggestions are especially effective for the detection of the so – called chiral superconductivity which is proposed to be realized, e.g., in  $\text{Sr}_2\text{RuO}_4$  [1–3] or heavy-fermions compounds [4, 5].

In particular, the nontrivial phase structure of the gap in the momentum space can be revealed by an external magnetic field which introduces vortex lines in the superconducting sample. The resulting inhomogeneous superconducting state can reveal many unusual magnetic and transport properties originated from the interplay between the nonzero vorticities in both the momentum and coordinate spaces. This interplay is known to be responsible for specific structure of quasiparticle subgap states inside the vortex cores investigated, e.g., in Ref [6]. Unfortunately, the key difference in the quasiparticle spectra of the vortex core states from the standard Caroli–de Gennes–Matricon (CdGM) ones appears only beyond the quasiclassical consideration. Namely, the anisotropic gap structure can shift the quasiparticle energy quantization rules causing the changes in the minigap value. Taking the simplest gap in the form  $p_x \pm ip_y$  we can get a complete suppression of the minigap [6]. The resulting zero energy mode appears to be extremely robust to the perturbations such as the ones caused by scattering centers, etc. [6]. However, the interlevel energy distances inside the cores are the order of  $\Delta_0^2/E_F$ , where  $\Delta_0$  is the superconducting gap magnitude and  $E_F$  is Fermi energy, which is very small value and the observation of the levels demands high resolution experimental techniques. The

existing scanning tunneling microscopy (STM)/scanning tunneling spectroscopy (STS) data, for instance, can not provide the reliable evidence for the minigap existence.

In this paper we suggest an alternative way to detect the chiral superconductivity in the vortex state studying the distinctive features of the vortex electronic structure in the presence of rather large columnar defects parallel to the applied magnetic field so that vortex is pinned over the entire length. Such defects can be created artificially by proton or heavy ion irradiation, by normal particles and nanorods inclusion and by introducing arrays of submicrometer holes. The quasiparticle spectrum in the vortex trapped by the columnar defect was studied in the Refs. [7] and [8] for the simplest case of conventional  $s$ -wave superconductor. It was shown that the defect leads to significant minigap increase up to the values of the order  $\Delta_0$ . One can expect that for unconventional superconductors the physical picture of the defect influence on the vortex states can become much more complicated and spectacular since in this case the subgap states bound to the defects or the sample surface can appear even without any vortices [9, 10]. These edge states should interact with the vortex bound states changing, thus, the resulting spectrum significantly. The hybridization of these quasiparticle modes has been studied previously for a particular case of a mesoscopic disc trapping a singly-quantized vortex [11]. It was shown that the vortex and antivortex spectra differ qualitatively due to the interplay between the internal vorticity in the momentum space and the vorticity in the coordinate space associated with the vortex.

Besides its fundamental interest the problem of pinned vortex spectrum in type-II superconductors with anisotropic gap is particularly important for understanding the nature of dissipation in such compounds. Indeed, vortex pinning has an influence on the vortex motion and, thus, strongly affects the superconductor transport properties in the flux flow regime. The microscopic consideration of pinned vortex matter should become important, of course, either at low temperatures well below the critical one or for defect dimensions smaller than the co-

herence length  $\xi$  (see discussion in Refs. [12–15]). The opposite limits corresponding to the temperature range close to the critical temperature or large defects can be perfectly described within phenomenological approaches [16–20].

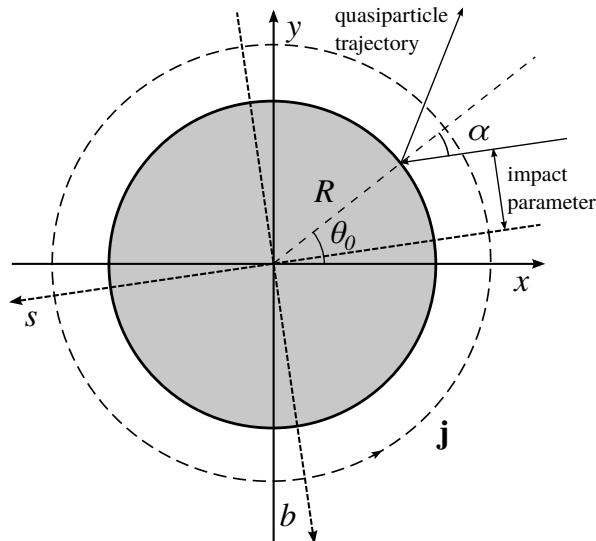


Figure 1. Quasiparticle specular reflection at the defect surface. Here  $s$  and  $b$  are the coordinates chosen along and perpendicular to the classical trajectory.

In this paper we do not consider, of course, the full problem of collective pinning which can be resolved only taking account of the vortex – vortex interaction. Instead we focus on the consideration of the individual pinned vortex and analyze the modification of the quasiparticle spectrum caused by the columnar defect assumed to have the form of an insulating cylinder of a finite radius  $R < \xi$  (see Fig. 1). We study the transformation of the anomalous energy branches originated from the normal reflection at the defect boundary. Let us elucidate the key point of our work and start from a qualitative discussion of the spectrum transformation. We analyze the spectrum within the one-dimensional quasiclassical quantum mechanics of the electrons and holes propagating along the classical trajectories. Each trajectory is defined by the impact parameter  $b$  and the trajectory orientation angle  $\theta_k$ . We assume the reflection to be specular and the trajectory experiencing the break at the defect consists of two rays with the same impact parameters and different  $\theta_k$  values satisfying the Snellius law. According to the general theory [21] the quasiparticle spectrum at each trajectory is determined by the order parameter phase difference  $\delta\varphi$  at the ends of the trajectory. This phase difference has two contributions arising from the vorticities in the coordinate and momentum spaces:  $\delta\varphi = \delta\varphi_r + \delta\varphi_k$ . The first term  $\delta\varphi_r = \mp 2 \arcsin(b/R)$  originates from the vortex phase in the coordinate space and the signs  $-$  and  $+$  correspond to opposite vorticities. The second term results from the order parameter phase circulation in the momentum space and for the simplest choice of the chi-

ral  $p$ -wave gap function  $\Delta \propto e^{i\theta_k}$  this contribution takes the form:  $\delta\varphi_k = -2 \arcsin(b/R) - \pi$ . Using the standard expression  $\epsilon_J(b) = -\Delta_0 \cos(\delta\varphi/2) \text{sign}[\sin(\delta\varphi/2)]$  for the subgap quasiparticle energy level in a single mode Josephson junction [21] we find:  $\epsilon_J(b) = -2\Delta_0 \text{sign}(1 - 2b^2/R^2) b/R\sqrt{1 - b^2/R^2}$  and  $\epsilon_J(b) = 0$  for the coinciding and opposite vorticities in the coordinate and the momentum spaces, respectively. Let us denote these vortices as  $N_+$  vortex and  $N_-$  vortex, correspondingly. The above result illustrates the qualitative difference between the chiral and conventional superconductors. For the latter case the total phase difference  $\delta\varphi$  is completely determined by  $\delta\varphi_r$  and, thus, the change of vorticity sign cannot change the energy levels.

The quasiparticles with large impact parameters  $|b| > R$  do not experience the reflection at the defect surface, so the spectrum in this case should be the same as the CdGM spectrum. Provided this crossover occurs at small  $b \ll \xi$  we get  $\epsilon_{CdGM}(b) \approx \Delta_0 b/\xi$  while in the large  $b \gtrsim \xi$  limit the spectrum saturates at the gap value [22]. The above reasoning cannot give this crossover to the CdGM branch because the Doppler shift associated with the superfluid velocity has been omitted. One can expect that in the limit  $R \ll \xi$  and  $b < R$  this Doppler shift contribution can not exceed the value of the CdGM energy  $\sim \Delta_0 b/\xi$ . Thus, for  $N_-$  vortices the spectrum should only slightly deviate from the Fermi level being close to the CdGM solution. In the case of  $N_+$  vortices the Doppler shift correction is small comparing to the term  $-2\Delta_0 b/R$  introduced above. The negative sign in the latter expression results in the important spectrum peculiarity: the inversion of the anomalous branch slope for the  $N_+$  vortex pinned by the defect comparing to the slope for a free vortex. Note that this inversion effect has been overlooked in numerical calculations presented in Ref. [23]. The change in the slope sign can cause rather drastic changes in the measurable characteristics. In particular, according to the spectral flow theory [24] it is the behavior of the anomalous branch which determines the high-frequency conductivity and Hall effect. In this paper we discuss possible influence of such spectrum transformation on the transport properties of the vortex state at finite frequencies.

The rest of the paper is organized as follows. In Section II the basic equations used for the spectrum calculation are introduced. In Sec. III we find the quasiparticle spectrum for a single-quantum vortex trapped by a columnar defect. The Sec. IV is devoted to the analysis of the local density of states (DOS). In the next Sec. V we discuss the defect influence on the high-frequency field response. The results are summarized in the last Sec. VI.

## II. MODEL

We consider a columnar defect as an insulator cylinder of the radius  $R$ . The magnetic field  $\mathbf{B}$  is parallel to the cylinder axis  $z$ , the vortex axis coincides with the

cylinder axis. Thus, the system is invariant with respect to the translations along the  $z$ -axis and the rotations around it. For simplicity we restrict ourselves with a two-dimensional case and consider a motion of quasiparticles only in the  $(x, y)$  plane. The excitation spectrum can be obtained from the Bogolubov-de-Gennes equations (BdG) written for a two-component quasiparticle wave function  $\psi(\mathbf{r}) = (u, v)$ :

$$-\frac{\hbar^2}{2m}(\nabla^2 + k_F^2)\tau_3\psi + (\hat{\Delta}(\mathbf{r})\tau_+ + h.c.)\psi = \epsilon\psi, \quad (1)$$

where  $\tau_{\pm} = (\tau_1 \pm i\tau_2)/2$ ,  $\tau_1$ ,  $\tau_2$  and  $\tau_3$  are the Pauli matrices in the Nambu space,  $\hbar k_F$  is the Fermi momentum,  $\hat{\Delta}$  is the superconducting gap operator. Considering an extreme type-II superconductor with a large London penetration depth  $\lambda \gg \xi$  we neglect the vector potential of the magnetic field  $A_{\theta} \approx Br/2$  because its contribution to the superfluid velocity  $A/\Phi_0 \propto r/\lambda^2$  is small compared to the gradient of the order parameter phase  $\propto 1/r$ [25]. Here  $r$  and  $\theta$  are the polar coordinates and  $\Phi_0$  is the magnetic flux quantum. We assume that the quasiparticle wavefunction does not penetrate the defect and imply the zero boundary conditions at the defect surface:

$$\psi(R, \theta) = 0. \quad (2)$$

### A. Quasiclassical approach

The superconducting gap  $\hat{\Delta}$  varies at the spatial scale  $\xi$  which is much greater than the atomic scale  $k_F^{-1}$  in all known superconductors. Hence one can solve the system (1) within the quasiclassical approximation. We follow the approach described in Refs. [7, 24] and introduce the momentum representation:

$$\psi(\mathbf{r}) = \int \frac{d^2k}{(2\pi)^2} e^{i\mathbf{k}\mathbf{r}} \psi(\mathbf{k}), \quad (3)$$

where  $\mathbf{k} = k(\cos\theta_k, \sin\theta_k) = k\mathbf{k}_0$ . The unit vector  $\mathbf{k}_0$  which depends on the angle  $\theta_k$  determines the trajectory direction in the  $(x, y)$  plane. We assume that the solutions of the equation (1) correspond to the absolute momentum values close to  $\hbar k_F$ :  $k = k_F + q$  ( $q \ll k_F$ ). Using the Fourier transformation

$$\psi(\mathbf{k}) = \frac{1}{k_F} \int_{-\infty}^{+\infty} ds e^{i(k_F - k)s} g(s, \theta_k). \quad (4)$$

One can finally express the wavefunction in the coordinate space  $\psi(\mathbf{r})$  through the function  $g(s, \theta_k)$ :

$$\psi(r, \theta) = \int_0^{2\pi} e^{ik_F r \cos(\theta_k - \theta)} g[r \cos(\theta_k - \theta), \theta_k] \frac{d\theta_k}{2\pi}. \quad (5)$$

To obtain the quasiclassical equation along the quasiparticle trajectory we introduce an angular eikonal:

$$g(s, \theta_k) = e^{iS(\theta_k)} \bar{\psi}(s, \theta_k), \quad (6)$$

assuming  $\bar{\psi}$  to be a slowly varying function of  $\theta_k$ . Quasiparticles propagating along the trajectories are characterized by the angular momentum  $\mu$ :

$$\mu = -k_F b = \frac{\partial S}{\partial \theta_k}. \quad (7)$$

Here we introduce the impact parameter  $b$  which determines the classical trajectory along with the  $\theta_k$  angle. The angular momentum is conserved due to the axial symmetry of the system.

Substituting (5) and (6) into (1) one can obtain the equation for  $\bar{\psi}$  at the classical trajectory (see Fig. 1):

$$-i\hbar v_F \tau_3 \frac{\partial \bar{\psi}}{\partial s} + (\tau_+ \bar{\Delta}(\mathbf{r}, \theta_k) + h.c.) \bar{\psi} = \epsilon \bar{\psi}, \quad (8)$$

where  $mv_F = \hbar k_F$ ,  $\bar{\Delta}$  is the quasiclassical form of the gap operator, and  $s$  is a coordinate along the classical trajectory. The transition from the  $(s, b)$  coordinates (see Fig. 1) to the usual cartesian  $(x, y)$  coordinates can be performed as follows:

$$x = s \cos \theta_k - b \sin \theta_k, \quad y = s \sin \theta_k + b \cos \theta_k. \quad (9)$$

Further consideration requires an explicit expression of the gap operator  $\hat{\Delta}$ .

In the homogeneous case the gap operator for the superconductors with spin-triplet coupling is determined by a momentum – dependent vector  $\mathbf{d}$  [26]:

$$\hat{\Delta}(\mathbf{k}) = -i\sigma_y(\mathbf{d}(\mathbf{k}), \sigma) = \begin{pmatrix} -d_x(\mathbf{k}) - id_y(\mathbf{k}) & d_z(\mathbf{k}) \\ d_z(\mathbf{k}) & d_x(\mathbf{k}) - id_y(\mathbf{k}) \end{pmatrix}. \quad (10)$$

We consider a superconductor with  $d_x = d_y = 0$  and  $d_z \propto \hat{k}_x + i\hat{k}_y$ ,  $\hat{k}_x = \hat{k}_x$ . Such an order parameter possibly describes a superconducting state in  $\text{Sr}_2\text{RuO}_4$  [27] and heavy-fermion compounds [28]. Separating the spin dependence and generalizing the operator for the inhomogeneous case we get the following expression for  $\Delta$ :

$$\hat{\Delta} = \frac{\Delta_0}{2k_F} (\{\eta_+(\mathbf{r}), -i\partial_+\} + \{\eta_-(\mathbf{r}), -i\partial_-\}) , \quad (11)$$

where  $\Delta_0$  is a gap magnitude,  $\eta_{\pm}(\mathbf{r})$  are the coordinate depending order parameters, which describe the Cooper pairs with the opposite angular momenta directions,  $\{a, b\} = ab + ba$  is an anticommutator,  $\partial_{\pm} = \partial/\partial x \pm i\partial/\partial y$ . Two degenerate ground states are described by the following order parameters:  $\eta_+ = 1$ ,  $\eta_- = 0$  and  $\eta_- = 1$ ,  $\eta_+ = 0$ . In a general inhomogeneous case both order parameters are non zero while usually far from the topological defects one of them is suppressed. The areas where only one order parameter is nonzero are called chiral domains.

One can find the quasiclassical form of  $\hat{\Delta}$ , neglecting the terms of the order of  $(k_F\xi)^{-1}$ :

$$\bar{\Delta}(\mathbf{r}, \theta_k) = \Delta_0 (\eta_+(\mathbf{r})e^{i\theta_k} + \eta_-(\mathbf{r})e^{-i\theta_k}). \quad (12)$$

Hence the equation (8) takes the following form:

$$-i\xi\tau_3\frac{\partial\bar{\psi}}{\partial s} + (D(\mathbf{r},\theta_k)\tau_+ + h.c.)\bar{\psi} = \varepsilon\bar{\psi}, \quad (13)$$

where  $\xi = \hbar v_F/\Delta_0$  is the coherence length,  $D(\mathbf{r},\theta_k) = \eta_+(\mathbf{r})\exp(i\theta_k) + \eta_-(\mathbf{r})\exp(-i\theta_k)$ ,  $\varepsilon = \epsilon/\Delta_0$ .

The axially symmetric vortex solutions are described by the following order parameters [29]:

$$\eta_{\pm}(\mathbf{r}) = f_{\pm}^{(m)}(r)e^{i(m\mp 1)\theta}, \quad (14)$$

where  $m$  is the sum of the winding numbers in the coordinate and the momentum spaces. One of the functions  $f_{\pm}$  saturates at unity at large  $r \gg \xi$  and another one vanishes far from the core. The magnetic flux carried by the vortex is determined by the winding number of the dominating order parameter component, i.e. it is equal to  $m+1$  flux quanta for a vortex trapped in the  $\eta_-$  domain and  $m-1$  flux quanta for a vortex in the  $\eta_+$  domain. Using (9) and (14) we find the order parameter profile at the classical trajectory:

$$D_b(s, \theta_k) = e^{im\theta_k} \sum f_{\pm}^{(m)} \left( \sqrt{s^2 + b^2} \right) \left( \frac{s + ib}{\sqrt{s^2 + b^2}} \right)^{m\mp 1}. \quad (15)$$

We can separate the  $\theta_k$  dependence, introducing a new function:

$$\bar{\psi} = e^{i(m\tau_3/2)\theta_k} \tilde{\psi} \quad (16)$$

and the equation (13) becomes:

$$-i\xi\tau_3\frac{\partial\tilde{\psi}}{\partial s} + \left( \tilde{D}_b(s)\tau_+ + h.c. \right) \tilde{\psi} = \varepsilon\tilde{\psi} \quad (17)$$

where  $\tilde{D}_b(s) = \exp(-im\theta_k)D_b(s, \theta_k)$ . For definiteness we consider the  $\eta_+$  domain, thus, the values  $m=2$  and  $m=0$  correspond to  $N_+$  and  $N_-$  vortices, respectively.

The true quasiparticle wavefunction  $\psi(k, \theta_k)$  must be, of course, a single-value function of  $\theta_k$ . This requirement imposes a certain quantization rule: the value of  $\mu + m/2$  must be an integer number. Since  $m$  is even for the singly-quantized vortex, the angular momentum  $\mu$  is an integer (cf. Ref. [6]). It means that defect does not change the quantization rule and does not shift the zero energy level.

### B. Boundary conditions

Now we need to rewrite the boundary condition (2) imposed on the quasiparticle wavefunction in the quasiclassical form. Using the expressions (5) and (16) we obtain:

$$\int_0^{2\pi} e^{ik_F R \cos \alpha + i\mu\alpha} e^{im\tau_3\alpha/2} \tilde{\psi}(R \cos \alpha) d\theta_k = 0. \quad (18)$$

Supposing the argument of the first exponent to vary rather fast we can use the stationary phase method in order to evaluate the integral. The stationary phase points are given by the equation  $\sin \alpha = \mu/k_F R = -b/R$ . This equation has no solutions if the impact parameter is greater than the defect radius. It means that the integral (18) is negligible and no boundary conditions are required because the trajectory does not hit the defect. In the opposite case  $|b| < R$  there are two stationary angles  $\alpha_1 = -\arcsin(b/R)$  and  $\alpha_2 = \pi - \alpha_1$  that correspond to the incident and reflected classical trajectories. The sum of the two contributions provides the boundary condition:

$$e^{i\varphi_0} \tilde{\psi}(s_0) = e^{-i\varphi_0} \tilde{\psi}(-s_0), \quad (19)$$

where  $s_0 = \sqrt{R^2 - b^2}$  and

$$\varphi_0 = k_F s_0 + (\mu + m\tau_3/2)(\alpha_1 - \alpha_2)/2 + \pi/4.$$

## III. EXCITATION SPECTRUM

### A. Large impact parameters $|b| > R$

In this case the trajectory does not hit the defect and does not experience any reflection, thus, the spectrum should be described by the standard CdGM solution [22].

Here we find spectrum and wave functions considering the imaginary part of  $\tilde{D}$  as a perturbation [30]. Neglecting the corresponding part in (17) we find:

$$-i\xi\tau_3\frac{\partial\tilde{\psi}}{\partial s} + \tau_1 \text{Re} \tilde{D}_b(s) \tilde{\psi} = \varepsilon\tilde{\psi}. \quad (20)$$

This equation has a zero eigenvalue with the following eigenfunction:

$$\tilde{\psi}_b = \frac{1}{\sqrt{2I_b}} \begin{pmatrix} i \\ 1 \end{pmatrix} e^{-K_b(s)}, \quad (21)$$

where

$$K_b(s) = \frac{1}{\xi} \int_0^s \text{Re} \tilde{D}_b(s') ds', \quad I_b = \int_{-\infty}^{+\infty} e^{-2K_b(s)} ds.$$

The first order perturbation theory yields the following excitation spectrum:

$$\varepsilon_b = \frac{1}{I_b} \int_{-\infty}^{+\infty} \text{Im} \tilde{D}_b(s) e^{-2K_b(s)} ds. \quad (22)$$

### B. Small impact parameters $|b| < R$

In this case the quasiparticle experiences reflection from the cylinder surface which modifies the spectrum.



In order to solve the equation (17) we have to take into account the boundary conditions (19), so we introduce the function:

$$\Psi(s) = \begin{cases} e^{+i\varphi_0} \tilde{\psi}(s), & s > 0 \\ e^{-i\varphi_0} \tilde{\psi}(s), & s < 0 \end{cases} \quad (23)$$

Due to the boundary condition (19)  $\Psi(s_0) = \Psi(-s_0)$ . The new function satisfies the following equation:

$$-i\xi\tau_3 \frac{\partial \Psi}{\partial s} + (\tau_+ G_b(s) + h.c.) \Psi = \varepsilon \Psi, \quad (24)$$

where  $G_b(s) = e^{i\phi \text{sign } s} \tilde{D}_b(s)$ ,  $\phi = m(\alpha_1 - \alpha_2)/2$ . The equation (24) is similar to a quasiclassical equation describing a Josephson junction: the order parameter is constant if  $s \rightarrow \pm\infty$ . Assuming such step like form of the order parameter along the trajectory we find the energy [21]:

$$\varepsilon = \chi \cos(\phi + \pi/2) = -\chi \sin \phi, \quad (25)$$

where  $\chi = \text{sign}(\cos \phi)$ . The energy depends only on the order parameter phase difference on the trajectory ends. The additional phase difference  $\pi$  arises from the order parameter symmetry property  $G_b(s) = -G_b^*(s)$ . The above approximate solution can be, of course, improved if we take account of the Doppler shift of quasiparticle energy caused by the superflow around the core. Such improvement is particularly important for the case of  $N_-$  vortex when the expression (25) yields  $\varepsilon = 0$  for all impact parameters and, thus, does not allow to get the correct slope of the anomalous spectral branch.

We can apply the perturbation theory used above in order to obtain a more precise solution. First we neglect the imaginary part of  $G$  and obtain the wave functions corresponding to the zero energy  $\varepsilon = 0$ :

$$\Psi_b(s) = \frac{1}{\sqrt{2I_b}} \begin{pmatrix} i\chi \\ 1 \end{pmatrix} e^{-K_b(s)}, \quad s > s_0, \quad (26)$$

where

$$K_b(s) = \frac{\chi}{\xi} \int_{s_0}^s \text{Re } G_b(s') ds', \quad I_b(s) = 2 \int_{s_0}^{+\infty} e^{-2K_b(s)} ds.$$

The eigenfunction is even,  $\Psi_b(s) = \Psi_b(-s)$ . This localized solution can be used as a zero-order approximation for the wave function. Within the first-order perturbation theory we find the spectrum:

$$\varepsilon_b = \frac{2\chi}{I_b} \int_{s_0}^{+\infty} \text{Im } G_b(s) e^{-2K_b(s)} ds. \quad (27)$$

The behavior of the subgap spectral branches found within this perturbation procedure is illustrated in Fig. 2(a) and (b) for  $N_+$  and  $N_-$  vortices, respectively. To verify the approximate solution we have also solved

the quasiclassical equations (17) numerically. The results of numerical calculations shown in Fig. 2 demonstrate a good coincidence with the ones obtained using the perturbation approach except the energies close to the superconducting gap  $\Delta_0$ . The failure of the perturbation procedure in this limit arises from the divergence of the wave function (26) localization radius.

The spectrum of the  $N_-$  vortex only slightly differs from the CdGM solution (see Fig. 2(b)). In this vortex the order parameter vorticity in  $\mathbf{r}$ -space is compensated by its chirality in  $\mathbf{k}$ -space and the phase difference at the ends of every classical trajectory is always equal to  $\pi$ . The defect effectively changes the order parameter amplitude along the trajectory and modifies the spectrum.

In opposite, for the  $N_+$  vortex the phase difference at the ends of classical trajectory causes a significant spectrum modification even for small impact parameters (see Fig. 2(a)). As a result, the subgap spectrum consists of three branches. Within the perturbation approach these branches reveal themselves in the spectrum discontinuity at the points  $b = \pm R/\sqrt{2}$ , where perturbation theory is not applicable. One can observe this energy discontinuity even in the simplified expression (25) where  $\chi$  changes sign at the points  $b = \pm R/\sqrt{2}$ . There are two branches which transform into the CdGM branch at large  $|b| > R$  and approach the superconducting gap at small  $b$ . The similar spectral branches have been observed earlier in the spectrum of a pinned vortex in a  $s$ -wave superconductor [7]. In addition to these branches there is an almost linear branch that goes through the origin with the slope inversed with respect to the CdGM solution (cf. the introductory section). We propose that this branch corresponds to the edge states bound to the surface of the unconventional superconductor. The spectrum of these surface states can be easily found within the quasiclassical approach solving the equation (24) with  $D = \exp(i\theta_k)$  that corresponds to the homogeneous chiral domain. Performing the same calculations as we had done for a vortex, we will obtain the following spectrum:

$$\varepsilon_b = \begin{cases} -\frac{b}{R}, & |b| < R \\ -\text{sign } b, & |b| > R \end{cases} \quad (28)$$

This quasiparticle spectrum is very close to the anomalous spectrum branch in the Fig. 2(a), so we can claim that this branch corresponds to the surface states.

This anomalous branch with the inversed slope has been overlooked by the authors of Ref. [23]. The results of their numerical calculations are shown in the Fig. 3. Note that at large angular momenta the spectrum found in Ref. [23] transforms into the CdGM solution as expected and is close to our numerical solution of (17).

#### IV. LOCAL DENSITY OF STATES

As a next step we turn to the calculations of the local density of states (LDOS) which can be probed, e.g., in

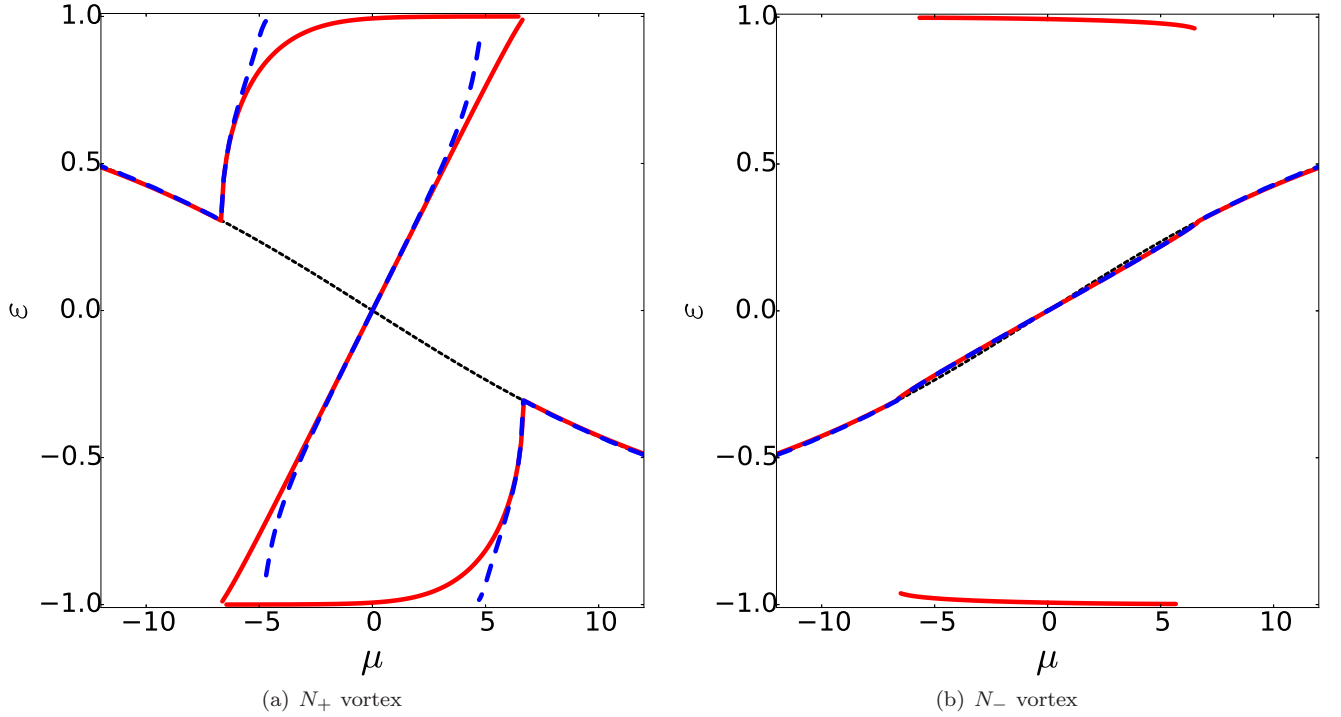


Figure 2. Quasiparticle spectrum for two vortex types found from the solution of the quasiclassical equation (17). The numerical solution is shown by the solid red lines, the dashed blue lines correspond to the results of perturbation theory, the black dashed line is the CdGM branch. The defect radius is  $R = 0.4\xi$ .

the STM/STS studies. The measurable quantity in these experiments is the local differential conductance:

$$\frac{dI/dV}{(dI/dV)_N} = \int_{-\infty}^{+\infty} \frac{N(\mathbf{r}, \epsilon)}{N_0} \frac{\partial f(\epsilon - eV)}{\partial V} d\epsilon, \quad (29)$$

where  $V$  is the voltage,  $(dI/dV)_N$  is the junction conductance in the normal state,  $N$  is the LDOS in the superconductor,  $N_0$  is the normal state DOS and  $f(\epsilon) = [1 + \exp(\epsilon/T)]^{-1}$  is the Fermi function. Within the quasiclassical approach the local DOS is determined as follows:

$$N(\mathbf{r}, \epsilon) = \frac{1}{2\pi} \int k_F |u_b(\mathbf{r})|^2 \delta(\epsilon - \epsilon_b) db. \quad (30)$$

Substituting (30) into (29) we obtain:

$$\frac{dI/dV}{(dI/dV)_N} = k_F \int_{-\infty}^{+\infty} \frac{|u_b(\mathbf{r})|^2}{N_0} \frac{\partial f(\epsilon_b - eV)}{\partial V} db. \quad (31)$$

The local DOS and the differential conductance are both expressed through the electron-like wave function  $u_b(\mathbf{r})$  corresponding to the energy  $\epsilon_b$ . We use the expressions (5), (6) and (16) in order to restore  $u_b(r)$ . If  $k_F r \gg 1$  it can be evaluated using the stationary phase method. In this limit the wave function is determined by

the quasiclassical wave functions at two classical trajectories passing through the point  $(r, \theta)$ :

$$u_b(r, \theta) = \frac{e^{i\mu\theta + im\theta/2}}{\sqrt{2\pi i k_F r}} \sum_{j=1,2} e^{i\varphi_j} \frac{\tilde{\psi}_{u,b}(r \cos \beta_j)}{\sqrt{\cos \beta_j}}, \quad (32)$$

where  $\varphi_j = k_F \cos \beta_j + i(\mu + m/2)\beta_j$ ,  $\beta_1 = -\arcsin b/R$ ,  $\beta_2 = \pi - \beta_1$ . Neglecting the part oscillating at the atomic length scale and applying the normalization condition we obtain:

$$|u_b(r)|^2 = \frac{\exp[-2K_b(\sqrt{r^2 - b^2})]}{\pi I_b \sqrt{r^2 - b^2}} \quad (33)$$

The local conductance is shown in the Fig. 4 for different types of vortices. The conductance profile for the  $N_-$  vortex (Fig. 4 b) reveals the typical CdGM behavior for  $r > R$  [31]. This conclusion is no more valid if we consider  $N_+$  vortex where the large slope of the inversed anomalous branch causes strong changes in the LDOS pattern (Fig. 4 a). The local conductance distribution in this case is similar to the one for a pinned vortex in the  $s$ -wave superconductor [7].

## V. HIGH-FREQUENCY CONDUCTIVITY

Besides the STM/STS studies there exists another efficient method for experimental investigation of quasiparticle subgap spectrum based on the measurements of

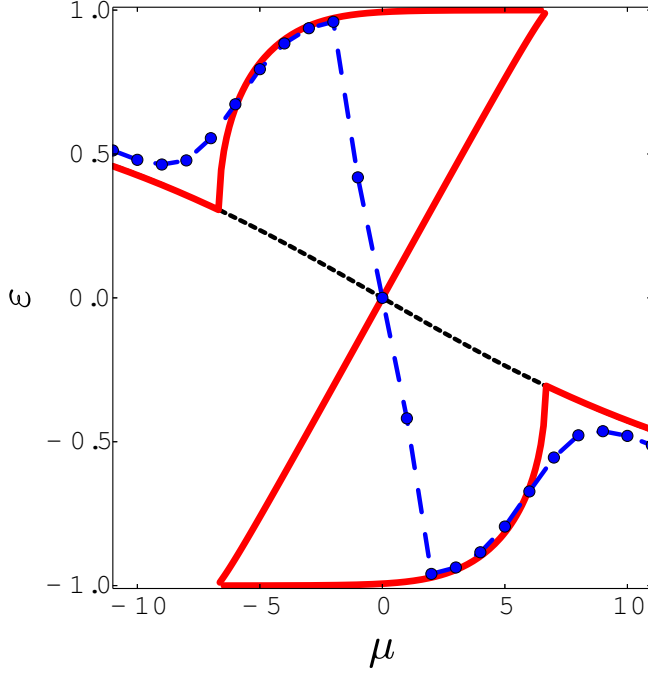


Figure 3. Comparison of the subgap spectral branches (red lines) in the  $N_+$  vortex with the ones found in Ref. [23] from numerical simulations on the basis of BdG theory (blue dashed lines).

the conductivity tensor at finite frequencies. In the classical limit the interaction of the quasiparticles with the high-frequency field can be described using the following Hamiltonian:

$$H(\mu, \theta) = \epsilon(\mu) + \hbar \mathbf{k}_F \mathbf{v}_s, \quad (34)$$

where  $\epsilon(\mu)$  is the energy of the anomalous spectral branch and  $\mathbf{v}_s$  is the superfluid velocity induced by the electromagnetic field. In London limit it is proportional to the vector potential  $\mathbf{v}_s = Q\mathbf{A}$ . Taking a circularly polarized field  $\mathbf{v}_s$  with frequency  $\Omega$ , finally we obtain the following Hamiltonian:

$$H(\mu, \theta) = \epsilon(\mu) + 2\hbar Q k_F \text{Re} (A_{\pm} e^{\pm i\theta - i\Omega t}), \quad (35)$$

where the sign “+” or “-” denotes the circular polarization orientation and  $A_{\pm}$  is the complex magnitude, i.e. the total magnetic potential is  $\mathbf{A} = \text{Re} (e^{-i\Omega t} A_{\pm} (\mathbf{x}_0 \pm i\mathbf{y}_0))$ . In order to find conductivity one should solve the Boltzmann equation written for the quasiparticle distribution function:

$$\frac{\partial f}{\partial t} + \frac{1}{\hbar} \left( \frac{\partial H}{\partial \mu} \frac{\partial f}{\partial \theta} - \frac{\partial H}{\partial \theta} \frac{\partial f}{\partial \mu} \right) = -\nu (f - f_0), \quad (36)$$

where  $f(\theta, \mu, t)$  is the distribution function,  $f_0(\mu)$  is the equilibrium distribution function and  $\nu$  is the quasiparticle relaxation rate. This equation can be solved within the perturbational approach, so that the total distribution function is represented as sum  $f = f_0 + f_1$ , where  $f_1$

is the first-order perturbation term:

$$f_1(\theta, \mu, t) = 2\text{Re} \frac{\pm i c Q k_F E_{\pm}}{\Omega (\Omega \mp \omega - i\nu)} \frac{\partial f_0}{\partial \mu} e^{\pm i\theta - i\Omega t}, \quad (37)$$

where  $\hbar\omega = \partial\epsilon/\partial\mu$  and  $E_{\pm} = i\Omega/cA_{\pm}$  is the electric field complex magnitude. Let us find the current for the zero-temperature case:

$$j_{\pm} = \frac{ecQk_F}{2\Omega (i\Omega \mp i\omega_0 + \nu)} E_{\pm}, \quad (38)$$

where  $\hbar\omega_0 = \partial\epsilon/\partial\mu|_{\mu=0}$ . One can easily obtain Ohmic and Hall conductivities from the Eq.(38):

$$\sigma_O = \frac{ecQ}{\Omega} \frac{\nu + i\Omega}{(\nu + i\Omega)^2 + \omega_0^2} \quad (39)$$

$$\sigma_H = -\frac{ecQ}{\Omega} \frac{\omega_0}{(\nu + i\Omega)^2 + \omega_0^2} \quad (40)$$

So one can see that the sign and the value of the Hall conductivity are strongly determined by the slope of the anomalous spectral branch at the Fermi level. The Hall conductivity can be probed experimentally, one of the methods is the polar Kerr effect measurements [3]. The following experiment can be proposed: the Hall conductivity can be measured at zero field and for the two opposite orientations of the magnetic field. According to the Eq.(40) the Hall conductivity for all three cases has the same sign. Since the slope of the spectral branch for the  $N_+$  vortex almost coincides with the one without vortex the Hall conductivity for this field orientation should coincide with the zero field Hall conductivity. Note that such coincidence is a fingerprint of the quasiparticle spectrum calculated above comparing to the one found in Ref. [23]. For the opposite field orientation the Hall conductivity appears to be suppressed due to the small spectral branch slope. Certainly, the above picture for the  $N_+$  vortex is valid for rather low frequencies  $\Omega < \Delta_0 R/\xi$ , i.e., when the rf field can not induce transitions to the levels at the broken CdGM branch in Fig.3.

## VI. SUMMARY

We have calculated the excitation spectrum in vortices pinned by columnar defects in chiral  $p$  wave superconductors. The spectrum is shown to depend strongly on the orientation of the magnetic field with respect to the internal angular momentum (chirality) of the Cooper pairs. If the magnetic field produce flux lines with the vorticity opposite to this internal angular momentum the quasiparticle spectra in pinned vortices are only slightly disturbed by the presence of defects. In the case of coinciding signs of vorticity and chirality the subgap spectra in pinned vortex cores appear to be strongly different from the ones for free vortices: the anomalous branch at small impact parameters changes its slope resulting in the change in the LDOS pattern and contribution of the

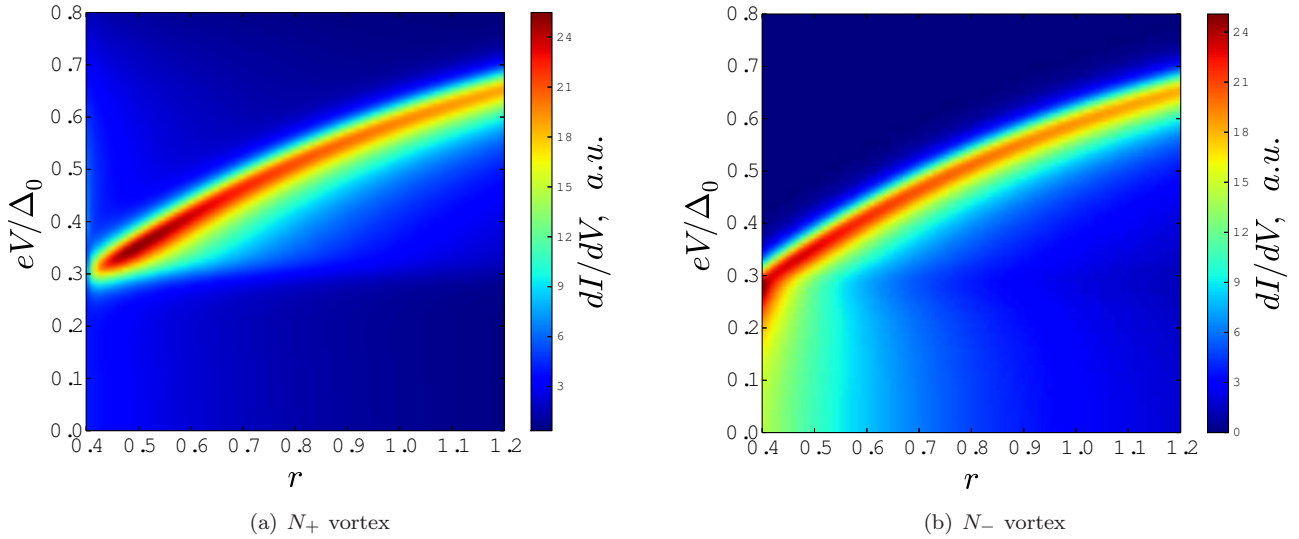


Figure 4. The local differential conductance vs the voltage  $V$  and the distance from the vortex axis  $r$  for different vortex types. Here we put  $R = 0.4\xi$ ,  $T = 0.02\Delta_0$ .

quasiparticles into the Ohmic and Hall conductivities at finite frequencies.

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